Teaching sound waves and sinusoids using Audacity and the TI-Nspire

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Abstract
The following exploration utilizes Audacity and TI-Nspire technology to explore the relationship between sound waves and trigonometry. In particular, students will learn how the ratio of frequencies in a musical interval determines the consonance or dissonance of the interval.

1. Introduction

The National Council of Teachers of Mathematics has stated that students are more likely to develop a profound and lasting understanding of mathematics if they make connections to different topics, contexts, and their own interests and experiences [2]. When teaching students how to graph trigonometric functions, I have found that the following lesson on sound waves helps students better appreciate the value and the precision required to graph more complicated sine and cosine waves.

2. Background

This lesson should take place after students have learned how to graph sine and cosine waves with varying amplitudes, frequencies, and horizontal and vertical shifts. Students should also know that the sine function is defined as $y = \sin(x)$, while a sinusoid function is a function of the form
$y = a \sin(bx + c)$.

3. Warmup

As a warmup, I give students two different tasks to complete without calculators. I ask that on the same coordinate grid, one half of the class graph $y = \sin x$ and $y = \sin 2x$, while the other half graph $y = \sin 5x$ and $y = \sin 7x$ (see Figure 1). After volunteers explain their work and take questions, I ask the class which pair of functions would be easier to graph, and there is general agreement that $y = \sin x$ and $y = \sin 2x$ is an easier problem. Some reasons that they offer include the following: (1) the periods for $y = \sin x$ and $y = \sin 2x$ are familiar numbers (2π and π, respectively); (2) the period of $y = \sin x$ is a multiple of the period of $y = \sin 2x$; and (3) both $y = \sin x$ and $y = \sin 2x$ complete a cycle at -2π and 2π. In addition, students often state that the ratio of the frequencies for $y = \sin x$ and $y = \sin 2x$ (2:1) is simpler than the ratio of the frequencies for $y = \sin 5x$ and $y = \sin 7x$ (7:5). While all of these suggestions are useful, this last concept is a key bridge to the main ideas in the lesson.

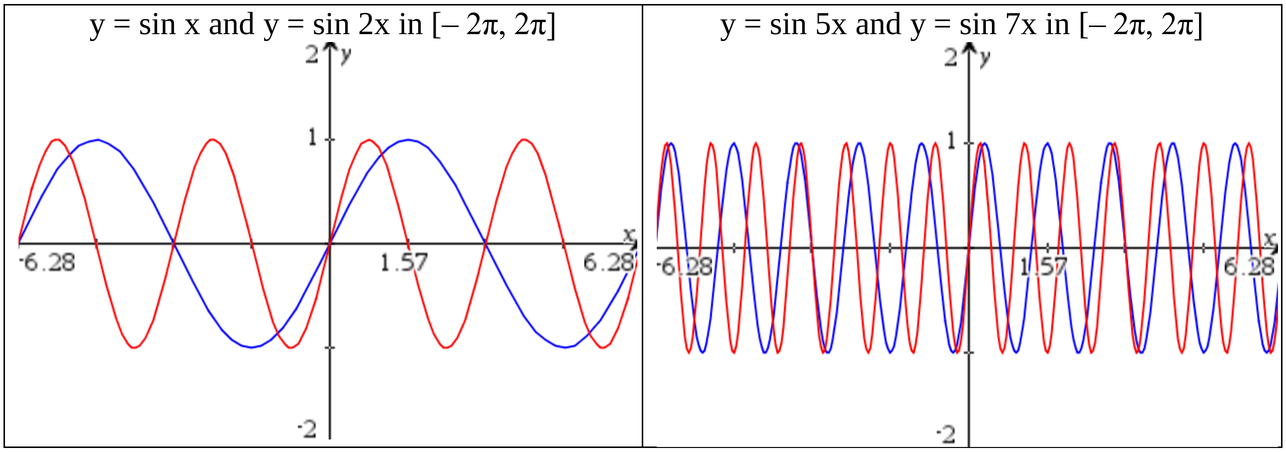


Figure 1. *Warmup (these graphs were created with TI-Nspire software)*

4. Single Tones

In order to be successful in this lesson, students need to understand that a single tone can be modeled by a sinusoidal wave. Additionally, students should know that the volume of the tone corresponds with the amplitude of the sinusoidal wave, and the pitch of the tone corresponds with the frequency of the sinusoidal wave (note: pitch is defined as “the location of a tone in relation to others, thus giving it a sense of being high or low” [4]). Teachers can present this information, elicit it from students, or use a program like Audacity to help students discover it for themselves (Audacity can be downloaded for free at audacity.sourceforge.net/download/). Depending on time, teachers may want to use Audacity, because it has the capability to play a tone and then graph the corresponding sinusoidal wave. For example, if you use the “generate” function in Audacity, you can create a tone with a frequency of 440 Hz on one page, and a tone with a frequency of 880 Hz on another. You can then play these tones and show students the corresponding graphs, and students will be able to recognize that the tone with a smaller frequency has a lower pitch, while the tone with a larger frequency has a higher pitch (see Figure 2).

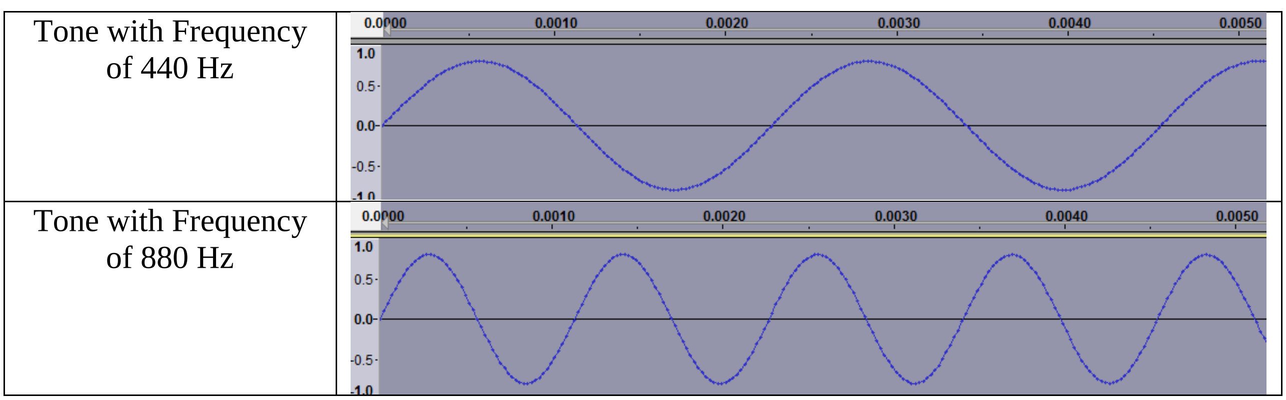


Figure 2. *Graphs of Tones with Different Frequencies (graphed with Audacity)*

In addition, the teacher can play tones with the same frequency, but different amplitudes, and students will be able to recognize that the tone with a larger amplitude produces a louder sound, while the tone with a smaller amplitude produces a softer sound (see Figure 3).

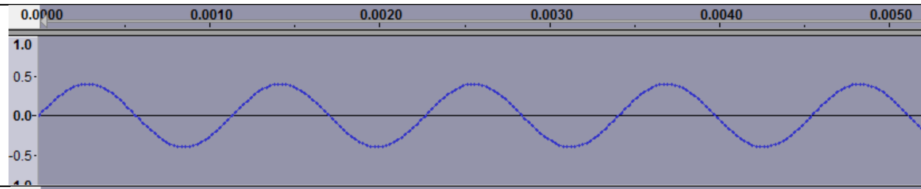
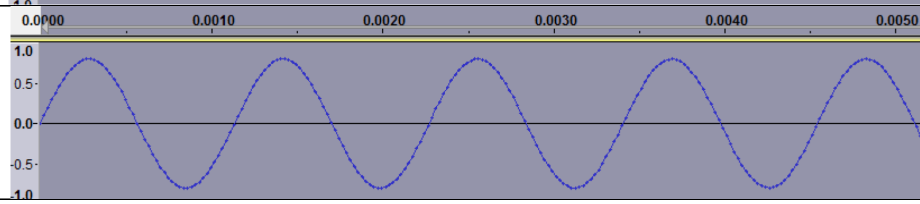
Tone with Amplitude of 0.4	
Tone with Amplitude of 0.8	

Figure 3. *Graphs of Tones with Different Amplitudes (graphed with Audacity)*

Once students discover that a single tone can be modeled by a sinusoidal wave, and further, that the amplitude of this wave determines the volume of the tone and the frequency of the wave determines the pitch of the tone, then the class can transition from an exploration of single tones to an exploration of intervals.

5. Intervals

An interval is defined as "the distance between two pitches" [4], but in order to demonstrate this concept to students, I either play various intervals on an online keyboard (such as http://www.onlinepianist.com/virtual_piano/), or I will ask students to sing the notes (with some practice before class) or play the notes on their instruments. We will play a) an octave (A – A), b) a minor second (A – B^b), c) a perfect fifth (A – E), and d) a perfect fourth (A – D), using A above middle C as the root. I ask students to discuss the consonance and dissonance of the intervals, and students are quick to agree that the minor second is dissonant, while the other three are consonant. Then, I ask groups of four to graph each of the intervals on their TI-Nspires (see Figure 4), and after playing each interval again, I ask each group to determine which interval corresponds with each graph (alternatively, you can distribute the graphs to each student). One benefit of asking students to create the graphs themselves is the fact that they have to agree on a standard window that allows them to analyze each graph.

1. Discussion

Students tend to make the following observations: (1) the pitches in Graph 2 tend to have function values with the same sign; (2) since the pitches in Graph 2 have similar frequencies, the graphs resemble a translation; and (3) with the exception of x = 0, the pitches in Graph 2 have no x-intercepts in common, while the pitches in the other three intervals do. Students with more musical training might argue that since Graph 2 has two pitches that have similar frequencies, the interval would create the dissonant sound of a minor second. At this point, I confirm that Graph 2 is the dissonant interval, and I reveal the interval that is associated with each graph, as well as the actual frequencies of the pitches (see Table 1).

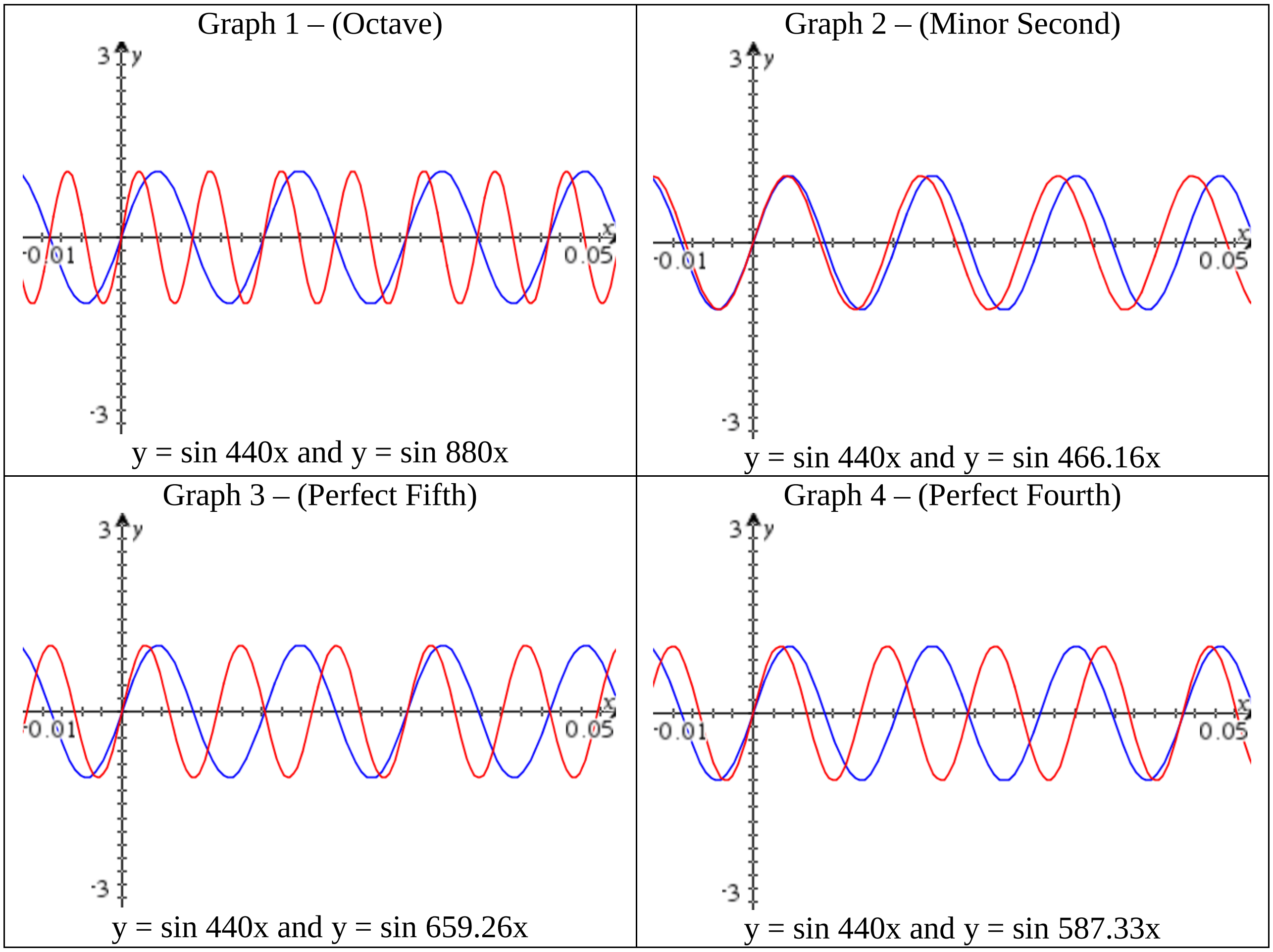


Figure 4. *Graphs of the Four Intervals in [– 0.01, 0.05] (created with TI-Nspire software)*

Once we match the intervals with the graphs, I ask students to consider the mathematics behind consonance and dissonance. Why is it that two pitches with frequencies of 440 Hz and 880 Hz sound consonant, while two pitches with frequencies of 440 Hz and 466.16 Hz sound dissonant? A student will normally suggest that the pitches in the octave have frequencies in a ratio of 1:2, while the pitches in the dissonant minor second do not have a similarly simple ratio. I will then ask students to test their theory on the other intervals, and a student will usually remark that the perfect fifth would have a simple ratio of 2:3 if the frequency of E was rounded to 660.

Table 1. *Four Musical Intervals and the Frequency of each Pitch*

Interval	Notes	Frequency of First Pitch (Hz)	Frequency of Second Pitch (Hz) (rounded to nearest hundredth)
Octave	A – A	440	880
Minor Second	A – B^b	440	466.16
Perfect Fifth	A – E	440	659.26
Perfect Fourth	A – D	440	587.33

The suggestion to round encourages students to look at the other intervals, and they will manipulate the numbers until they discover the 3:4 ratio of the perfect fourth and the 15:16 ratio of the minor second. They are amazed to discover that the consonance or dissonance of an interval is really just based on simple mathematics: namely, an interval is consonant if the ratio of the frequencies of its pitches can be expressed using small, whole numbers (see Table 2).

Table 2. *The Ratio of Frequencies for Four Musical Intervals*

Interval	Pitches	Frequency of First Pitch (Hz)	Frequency of Second Pitch (Hz) (rounded to nearest hundredth)	Ratio of Frequencies
Octave	A – A	440	880	1:2
Minor Second	A – B^b	440	466.16	≈ 15:16
Perfect Fifth	A – E	440	659.26	≈ 2:3
Perfect Fourth	A – D	440	587.33	≈ 3:4

At this point, if a student hasn't already done so, I reference the warmup, and ask students to explain why we started with that seemingly unrelated problem. Students may remember that it was easier to graph waves with a simpler ratio of frequencies, so this may convince them that these intervals are not only easier to graph, but they are also easier to process aurally, which results in a consonant sound.

2. Follow-up Activities

1. Look at actual frequencies

Students can look at actual frequencies of notes, with the goal of determining pitches that would create a consonant or dissonant sound (frequencies can be found at http://en.wikipedia.org/wiki/Piano_key_frequencies). For example, students will notice that the higher pitch in every octave has double the frequency of the lower pitch, while other intervals will always have approximate ratios of 2:3 (perfect fifths) or 3:4 (perfect fourths). Students can verify their discoveries using virtual keyboards or musical instruments in class.

2. Look at intervals using combined sinusoids

A second activity asks students to look again at consonance and dissonance, except this time with graphs of combined sinusoids, rather than two distinct waves. These graphs can add another layer to student understanding, since students will see that the consonant intervals have a fairly regular, repeating graph, while the minor second looks more like a single sine wave with a varying amplitude (see Figure 5).

The graphs of the combined sinusoids provide some insight into the phenomenon of beating. Beating is defined as a "throbbing that is heard when two notes are slightly out of tune" [4]. Students often hear something similar to this throbbing when the minor second is played, as they describe it as a single tone that varies in volume (for a more accurate exploration of throbbing, students can use electronic resources to listen to notes that are slightly out of tune, or student musicians can bend a pitch on their instrument or with their voices).

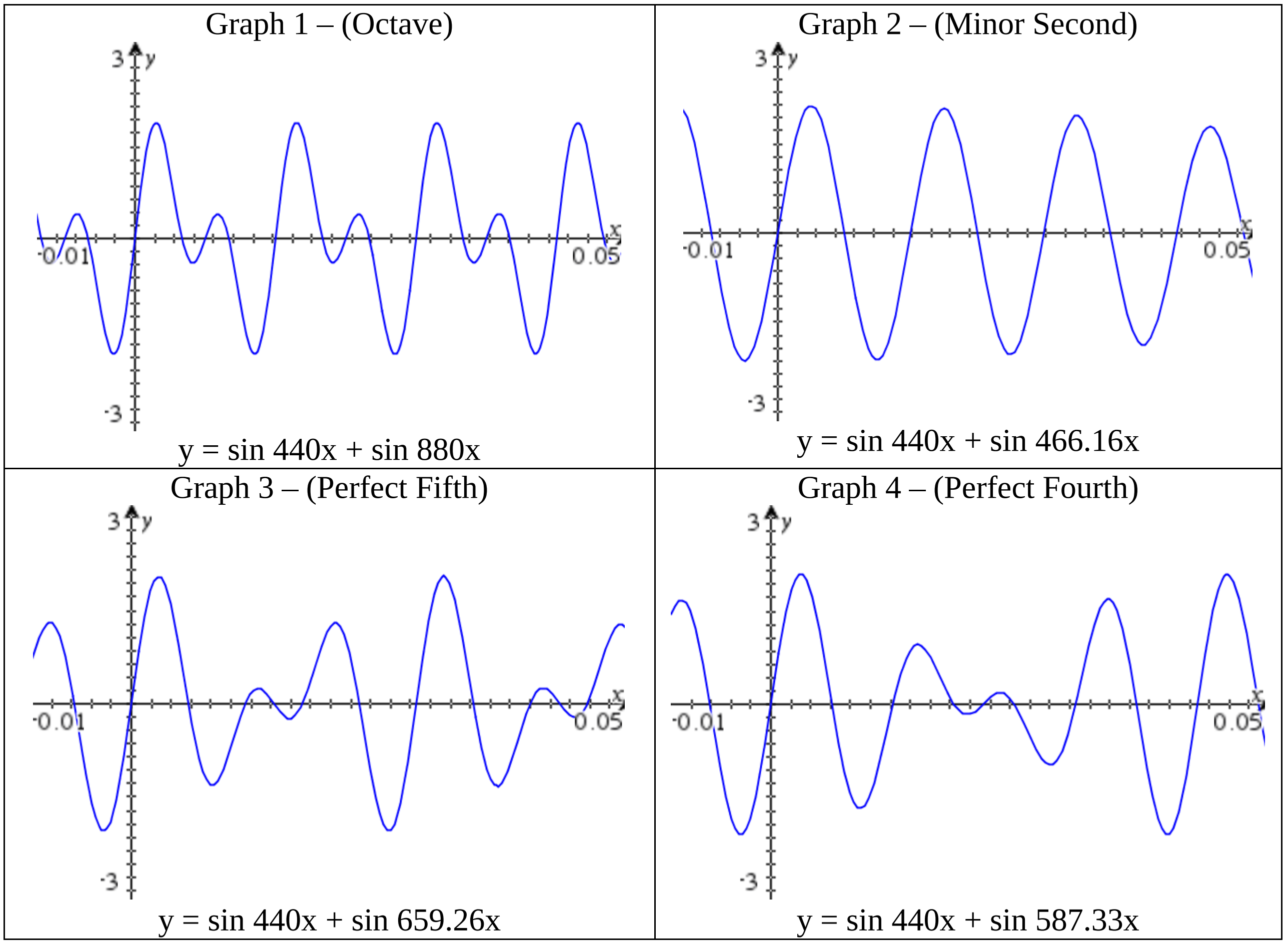


Figure 5. *Intervals Graphed as Combined Sinusoids [– 0.01, 0.05] (created with TI-Nspire)*

When looking at the graph of the minor second in Figure 5, students can relate what they hear to what they see, since they can visualize the minor second as a single sine wave (tone) with a varying amplitude (volume). Students can explore this algebraically by considering the behavior of the product side of the sum-to-product formula $\sin a + \sin b = 2\sin(\frac{a+b}{2})\cos(\frac{a-b}{2})$.

3. Graph sound waves using Audacity

If students are comfortable using Audacity, they can play different intervals and have the computer capture the sound and graph the corresponding waves. These intervals appear on Audacity as combined sinusoids, so students would need to be comfortable with the previous activity. This activity goes above and beyond, though, because it allows students to direct the activity by choosing their own intervals, hypothesizing whether they are consonant or dissonant, and then verifying their hypotheses graphically.

3. Conclusion

In conclusion, this article describes a lesson that looks at the relationship between trigonometry and sound waves. Throughout their engagement in the lesson, students utilize several of the Standards for Mathematical Practice, including modeling with mathematics, using appropriate tools strategically, and attending to precision [3]. These connections between trigonometry and music might seem like an interesting aside to some students, but for others, they represent an invitation to engage with abstract material and relate it to their everyday lives. I have found this lesson to be an effective way to not only engage students, but also to help them better understand and appreciate frequency and period.

4. Acknowledgments

The development of this lesson was a collaborative effort, and it relied on the wisdom of several friends and colleagues. In particular, Christian Smythe, Daniel Burwasser, Jenna Russo, and Jason McManus provided invaluable assistance, as did several colleagues in the Math for America program.

5. References

[1] Audacity Team (2012). Audacity(R) Version 2.0.0.
[2] National Council of Teachers of Mathematics (NCTM) (2000). *Principles and Standards for School Mathematics*. Reston, VA: NCTM.
[3] National Governors Association Center for Best Practices & Council of Chief State School Officers. (2010). *Common Core State Standards for Mathematics.* Washington, DC: Authors.
[4] Virginia Tech Department of Music. “Virginia Tech Multimedia Music Dictionary.” Accessed August 8, 2013. http://www.music.vt.edu/musicdictionary/.